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A COMPARISON OF FREE-SPACE AND FIBER MIXER PERFORMANCES

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This report compares the heterodyne reception mixing efficiency of a laser radar employing a conventional Free-Space mixer to one employing a fiber mixer. A rigorous derivation for speckle targets shows the two mixing schemes will have approximately the same mixing efficiency. The fiber mixing efficiency is optimized by choosing the optics numerical aperture (NA) equal to the fiber NA.

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TABLE OF CONTENTS

LIST OF FIGURES

FOREWORD

This Technical Report was prepared by the Electro-Optical Techniques Section, Mission Avionics Division, Wright Laboratory, Wright-Patterson AFB OH.

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A Comparison of Free-Space and Fiber Mixer Performances

I. Introduction

A conventional heterodyne detection system employs mixing between the signal field and the local oscillator (LO) fields in free space. In this system not all the optical power collected by the receiving aperture contributes to the power in the intermediate frequency (IF) signal. Only that portion of the received power which is in the same temporal and spatial mode as the LO will contribute to the IF signal [1].

In a heterodyne detection system employing a single mode fiber mixer, the signal and LO fields are perfectly matched, so all the signal power propagating in the fiber will contribute to the IF signal. However, not all the power focused onto the end of a single mode optical fiber will couple into the fiber's single propagating mode.

In order to compare a heterodyne system employing free space mixing between the LO and the received field with a heterodyne system employing mixing in a fiber coupler, we need to compare the IF carrier-to-noise ratio (CNR) from each system for identical system design parameters. In order to do this, we need to be able to compute what fraction of the received optical power actually couples into the fiber. The bulk of this report, and Section III in particular, attacks this computation.

The numerical results here indicate that for most reasonable radar optical designs, the free space mixer will do as well or better than the fiber mixer. However, the fiber mixer can perform as well as the free space mixer when the receiver optics f-number is approximately matched to the f-number of the fiber. This will often result in a reasonable choice for the receiver optics design.

II. Free Space Mixer Performance

The IF signal power from a free space mixing system is not difficult to calculate. All the equations necessary to make the computation are already well known and the following discussion merely collects those results into a form suitable to solve our specific problem.

We will match the local oscillator (LO) field (referenced to the transmitting aperture) to the transmitted field and define the fields at the transmitting aperture as:

$$\underline{U}_T(\overline{\rho},t) = \sqrt{P_T}\ \underline{s}(t)\ \underline{\xi}(\overline{\rho}), \qquad \underline{U}_{LO}(\overline{\rho},t) = \sqrt{P_{LO}}\ e^{j2\pi\nu_{IF}t}\ \underline{\xi}(\overline{\rho}) \tag{2-1}$$

where: P_T is the transmitted power,

P_{LO} is the LO power,

$\underline{\xi}(\overline{\rho})$ is the transmitted beam shape at the transmitter aperture plane, normalized to unit area (that is, $\int\int_{A_T} d\overline{\rho}\,|\underline{\xi}(\overline{\rho})|^2 \equiv 1$ where A_T is the area of the transmitting aperture),

$\underline{s}(t)$ is the transmitted pulse shape normalized to unity time average power (indicated by the notation $< \ >$):

$$<|\underline{s}(t)|^2> \equiv \frac{1}{T}\int_0^T dt\,|\underline{s}(t)|^2 \equiv 1 \text{ (where } T \text{ is the pulse width), and}$$

ν_{IF} is the IF frequency in Hz.

It is custom..y in much of the laser radar literature to write the detector IF current as a target plane integral by back propagating the LO field to the target [2]. Doing this, the signal component of the IF current out of the detector is [3]:

$$\underline{\widetilde{y}}(t) = \frac{q\eta}{h\nu_o}\int\int_{A_t} d\overline{\rho}\ \underline{\widetilde{T}}(\overline{\rho}, t-\tfrac{L}{c})\ \underline{U}_t(\overline{\rho}, t-\tfrac{2L}{c})\ \underline{U}^*_{lo}(\overline{\rho},t) \tag{2-2}$$

where: q is the electronic charge,

η is the detector quantum efficiency,

h is Planck's constant,

ν_o is the optical frequency,

λ is the wavelength of the light,

L is the range to the target,

c is the velocity of light, $c = \lambda\nu_o$,

A_t is the target area,

$\underline{\widetilde{T}}(\overline{\rho},t)$ is the target's random, complex reflectivity,

$\underline{U}_t(\overline{\rho}, t-\frac{2L}{c})$ is the far field pattern of the transmitter at the target plane, and

$\underline{U}^*_{lo}(\overline{\rho},t)$ is the local oscillator (LO) beam pattern back propagated to the target plane.

Using the Green's function for free space propagation and making the far field assumption $A_T \ll \lambda L$, the transmitter and back propagated LO fields at the target plane are approximately[1]:

$$
\begin{aligned}
\underline{U}_t(\overline{\rho},t) &\simeq \frac{e^{jkL}}{j\lambda L}\exp\left(-\frac{jk}{2L}|\overline{\rho}|^2\right)\mathcal{U}_T\left(\frac{\overline{\rho}}{\lambda L},t\right) \\
\underline{U}_{lo}(\overline{\rho},t) &\simeq \frac{e^{jkL}}{j\lambda L}\exp\left(-\frac{jk}{2L}|\overline{\rho}|^2\right)\mathcal{U}_{LO}\left(\frac{\overline{\rho}}{\lambda L},t\right)
\end{aligned}
\qquad (2-3)
$$

where $k \equiv 2\pi/\lambda$ and it must, of course, be true that:

$$
\begin{aligned}
\underline{\mathcal{U}}_T\left(\frac{\overline{\rho}}{\lambda L},t\right) &= \sqrt{P_T}\,\underline{s}(t)\mathcal{F}_{\overline{\rho}'}\{\underline{\xi}(\overline{\rho}')\}\Big|_{\frac{\overline{\rho}}{\lambda L}} \\
\underline{\mathcal{U}}_{LO}\left(\frac{\overline{\rho}}{\lambda L},t\right) &= \sqrt{P_{LO}}\;e^{j2\pi\nu_{IF}t}\mathcal{F}_{\overline{\rho}'}\{\underline{\xi}(\overline{\rho}')\}\Big|_{\frac{\overline{\rho}}{\lambda L}}.
\end{aligned}
\qquad (2-4)
$$

Substituting these quantities into equation $(2-2)$, we can write the IF signal component as:

$$
\widetilde{\underline{y}}(t) = \frac{q\eta}{h\nu_o}\frac{1}{(\lambda L)^2}\int\!\!\int_{A_t} d\overline{\rho}\;\widetilde{\underline{T}}(\overline{\rho},t-\tfrac{L}{c})\;\underline{\mathcal{U}}_T\left(\frac{\overline{\rho}}{\lambda L},t-\tfrac{2L}{c}\right)\underline{\mathcal{U}}^*_{LO}\left(\frac{\overline{\rho}}{\lambda L},t\right). \qquad (2-5)
$$

For a stationary, purely speckle target the target statistics are [3]:

$$
E[\widetilde{\underline{T}}(\overline{\rho}_1)] = 0, \qquad E[\widetilde{\underline{T}}(\overline{\rho}_1)\widetilde{\underline{T}}(\overline{\rho}_2)] = 0, \qquad E[\widetilde{\underline{T}}(\overline{\rho}_1)\widetilde{\underline{T}}^*(\overline{\rho}_2)] = \lambda^2\mathcal{T}_s(\overline{\rho}_1)\delta(\overline{\rho}_1-\overline{\rho}_2), \qquad (2-6)
$$

where $E[\,]$ indicates a statistical expectation (average) over the random variable or process and $\mathcal{T}_s(\overline{\rho})$ is related to the reflectance of the target[2] at coordinate $\overline{\rho}$. If the target is range unresolved, approximately stationary over T, and uniform over the area illuminated by the transmitted beam and the time lags (the $t-\frac{L}{c}$ and $t-\frac{2L}{c}$ effects) are negligible, we can use these statistics to find the mean, time-average, IF signal power:

$$
< E[|\,\widetilde{\underline{y}}(t)|^2] > = \left(\frac{q\eta}{h\nu_o}\right)^2 P_{LO}\frac{\lambda^2\mathcal{T}_s P_T}{(\lambda L)^4}\int\!\!\int_{A_t} d\overline{\rho}\left|\mathcal{F}_{\overline{\rho}'}\{\underline{\xi}(\overline{\rho}')\}\Big|_{\frac{\overline{\rho}}{\lambda L}}\right|^4. \qquad (2-7)
$$

But to complete this calculation, $\underline{\xi}()$ needs to assume a specific form.

Let the transmitted beam shape, $\underline{\xi}(\overline{\rho})$, be the usual Gaussian function with a $1/e^2$ beam power

---

[1] In this report, field functions in a script font indicate the Fourier transform of the respective field function in the Roman italic type. That is, $\underline{\mathcal{U}}_T(\overline{\omega},t) \equiv \mathcal{F}_{\overline{\rho}}\{\underline{U}_T(\overline{\rho},t)\}\big|_{\overline{\omega}}$ where the notation $\mathcal{F}_{\overline{\rho}}\{\underline{U}_T(\overline{\rho},t)\}\big|_{\overline{\omega}}$ means take the Fourier transform of the given function on variable $\overline{\rho}$ and evaluate it at the spatial frequency argument $\overline{\omega}$. In this case, $\underline{U}_T(\overline{\rho},t)$ is the field in the transmitter aperture plane. See Appendix I for more information on this notation and how it relates to other notations.

[2] For a uniform target, $\mathcal{T}_s$ is related to the commonly used diffuse target reflectivity, ρ, by $\mathcal{T}_s = \frac{\rho}{\pi}$ [9].

radius of w_o, normalized to unit area and let the transmitting aperture A_T be large enough that it does not appreciably truncate the transmitted beam or the LO. Then the the beam shape and its Fourier transform are:

$$\underline{\xi}(\bar{\rho}) = \sqrt{\frac{2}{\pi w_o^2}} \exp\left(-\frac{|\bar{\rho}|^2}{w_o^2} \right) \quad \Leftrightarrow \quad \mathcal{T}_{\bar{\rho}'}\{\underline{\xi}(\bar{\rho}')\}\Big|_{\frac{\bar{\rho}}{\lambda L}} = \sqrt{2\pi w_o^2} \exp\left(-\left(\frac{\pi w_o}{\lambda L}\right)^2 |\bar{\rho}|^2 \right). \qquad (2-8)$$

If the target is resolved by the transmitter aperture (*i.e.*, the illuminator beam extent at the target is less than the target area) the integral over the target is easy to do and the result gives:

$$< E[| \ \tilde{\underline{y}}(t)|^2] > = \left(\frac{q\eta}{h\nu_o}\right)^2 P_{LO} \lambda^2 \mathcal{T}_s P_T \cdot \pi \left(\frac{w_o}{\lambda L}\right)^2. \qquad (2-9)$$

To find the carrier-to-noise ratio[3] (CNR) out of the detector, assume the post-detection filter bandwidth, B_N, is greater than the bandwidth of the transmitted waveform $\underline{s}(t)$ and divide by the expected value of the LO shot noise component, $(q^2\eta/h\nu_o)P_{LO}B_N$ [1]:

$$\text{CNR}_{\text{free-space}} = \frac{\eta}{h\nu_o B_N} P_T \mathcal{T}_s \frac{\pi w_o^2}{L^2}. \qquad (2-10)$$

We will compare this with the CNR from a fiber mixing system, $\text{CNR}_{\text{fiber}}$, for identical system parameters B_N, P_T, $\mathcal{T}_s$, w_o, L, and A_T.

It is interesting to examine this CNR in the following manner. The first portion of the CNR expression, $\eta/h\nu_o B_N$, is just the inverse of the IF LO shot noise power. Therefore, the remainder of the CNR expression, $P_T \mathcal{T}_s \pi w_o^2/L^2$, is the IF signal power. However, the expected value of the total power collected by the receiving aperture is:

$$E[\tilde{P}_c] = P_T \cdot \frac{\rho}{\pi} \cdot \frac{A_R}{L^2} = P_T \mathcal{T}_s \frac{A_R}{L^2}. \qquad (2-11)$$

Obviously, only a portion of the power collected by the receiver aperture actually contributes to the IF CNR. Dividing the last two quantities, assuming a monostatic system (so $A_R \equiv A_T$), and using $w_o = d_T/4$, (the maximum usable value of w_o for which beam truncation is negligible), gives:

$$\frac{\pi w_o^2}{A_R} = \frac{1}{4}. \qquad (2-12)$$

In other words, in a free-space mixing system sensing a purely speckle target, only $\frac{1}{4}$ of the collected optical power actually contributes to improving the IF CNR. The remaining collected optical power is

[3]There is a difference between carrier-to-noise ratio (CNR) and signal-to-noise ratio (SNR). The CNR, as defined here, is sometimes (and incorrectly) referred to as the SNR. The SNR is the CNR reduced by the effect of variations in the received power due to target variations and atmospheric turbulence. The SNR is always less than or equal to the CNR.

in spatial modes which do not mix with the LO spatial mode. It is possible to improve on the percentage of power which contributes to the IF CNR by using a Gaussian beam with a w_o which is a larger fraction of d_T, but this leads to beam truncation, increased beam divergence in the far field, and the above equation no longer accurately predicts the coupling efficiency to the IF. Rye and Frehlich have shown, however, that even with the optimum choice of w_o for a given d_T, the maximum efficiency when using truncated Gaussian beams is limited to about 44% [10]. Similarly, using $d_T/w_o > 4$ will reduce the mixing efficiency to even less than $\frac{1}{4}$.

III. Fiber Mixer Performance

A. General analysis

Now compute the IF signal power for the fiber mixing system. Since the local oscillator and the received optical field will be propagating in the same single mode fiber, their field patterns must be identical. A single mode fiber will only propagate one field pattern, called the HE_{11} mode. Designate this normalized field pattern $\underline{U}_{11}(\overline{\rho})$. The signal component of the IF current out of the detector is:

$$
\begin{aligned}
\underline{\widetilde{y}}(t) &= \frac{q\eta}{h\nu_o} \int\int_{A_d} d\overline{\rho} \left(\sqrt{\widetilde{P}_f}\,\underline{U}_{11}(\overline{\rho})\,\underline{s}(t-\tfrac{2L}{c})\right)\left(\sqrt{P_{LO}}\,\underline{U}^*_{11}(\overline{\rho})\,e^{-j2\pi\nu_{IF}t}\right), \\
&= \frac{q\eta}{h\nu_o}\sqrt{P_{LO}\widetilde{P}_f}\;\underline{s}(t-\tfrac{2L}{c})\,e^{-j2\pi\nu_{IF}t}\int\int_{A_d} d\overline{\rho}\,|\underline{U}_{11}(\overline{\rho})|^2, \\
&= \frac{q\eta}{h\nu_o}\sqrt{P_{LO}\widetilde{P}_f}\;\underline{s}(t-\tfrac{2L}{c})\,e^{-j2\pi\nu_{IF}t}.
\end{aligned}
\qquad (3-1)
$$

where $\widetilde{P}_f$ is the received optical power coupled into and propagating in the fiber, A_d is the detector area (which is much larger than the mode field extent), and, once again, $\underline{U}_{11}(\overline{\rho})$ is normalized to unit area. $\widetilde{P}_f$ is a random variable because of target speckle. Consequently, the mean, time-average IF signal power and CNR are:

$$
< E[|\,\underline{\widetilde{y}}(t)|^2] > = \left(\frac{q\eta}{h\nu_o}\right)^2 P_{LO}\,E[\widetilde{P}_f] \;\Rightarrow\; \mathrm{CNR}_{\mathrm{fiber}} = \frac{\eta}{h\nu_o B_N}E[\widetilde{P}_f], \qquad (3-2)
$$

and all the received power coupled into the fiber contributes to the mean, time-average IF signal power. It now only remains to compute $E[\widetilde{P}_f]$.

To compute $\widetilde{P}_f$, propagate the transmitted field $\underline{U}_T(\overline{\rho})$ to the target using the free space Green's function,

$$
\underline{h}_L(\overline{\rho}) \equiv \frac{e^{jkL}}{j\lambda L}\exp\!\left(\frac{jk}{2L}|\overline{\rho}|^2\right), \qquad (3-3)
$$

and multiply by the target's complex reflectivity, $\underline{T}(\overline{\rho})$. Back propagate the resulting field to the receiving aperture using the same Green's function and then through the lens to its focal plane. This will give us the field at the lens' focal plane, $\underline{\widetilde{U}}_f(\overline{\rho}_f)$, in terms of the target and system characteristics. The end of the fiber will be at the lens' focal plane where it will collect whatever power it can from the field. Finding $\underline{\widetilde{U}}_f(\overline{\rho}_f)$ is not difficult. Using the free space Green's function, assuming far field operation ($A_T \ll \lambda L$), and ignoring insignificant quadratic phase terms, the result is (see Appendix II for a complete derivation of this formula):

$$\widetilde{\underline{U}}_f(\overline{\rho}_f) = \frac{1}{(j\lambda f)(\lambda L)^2} \iint_{A_t} d\overline{\rho}\, \widetilde{\underline{T}}(\overline{\rho}, t - \tfrac{L}{c})\, \underline{\mathcal{U}}_T(\tfrac{\overline{\rho}}{\lambda L}, t - \tfrac{2L}{c})\, \mathcal{W}_R\left(- \frac{\overline{\rho} - \frac{L}{f}\overline{\rho}_f}{\lambda L} \right) \qquad (3-4)$$

where: f is the focal length of the receiving lens,

$\underline{\mathcal{U}}_T(\frac{\overline{\rho}}{\lambda L}, t)$ is the Fourier transform of the beam in the transmitter aperture (which is proportional to the far field beam pattern of the transmitter at the target plane), and

$\mathcal{W}_R(\overline{\omega})$ is the Fourier transform of the receiving aperture function, $W_R(\overline{\rho})$. ($W_R(\overline{\rho})$ is 1 where the aperture is clear and 0 where it is opaque.)

In words, this formula represents an image of the target in the focal plane of the lens, with illumination given by the far field pattern of the transmitted beam, and with the image blurred by the diffraction introduced by receiving aperture's Fourier transform. (This is easiest to see by letting the receiving aperture be infinitely large so the function $\mathcal{W}_R()$ becomes a Dirac delta function.)

This is the field outside the fiber face. Inside the fiber face, the field is that of the HE_{11} mode of a dielectric waveguide. The field distribution of this mode is proportional to $J_0(r)$ in the fiber core (where r is the radial distance from the center of the fiber) and proportional to $K_0(r)$ in the fiber cladding, where $J_0()$ is the zero order Bessel function and $K_0()$ is the zero order modified Bessel function. Fortunately, however, Marcuse [4],[5] has shown that a Gaussian function very accurately approximates the field distribution. Therefore, the normalized single mode field distribution is approximately:

$$\underline{U}_{11}(\overline{\rho}) \simeq \sqrt{\frac{2}{\pi w^2}}\, \exp\left(- \frac{|\overline{\rho}|^2}{w^2} \right), \qquad (3-5)$$

where Marcuse also shows the optimum choice for the parameter w is given empirically by the equation:

$$w \equiv r_c g_o(V) = r_c \left(0.65 + \frac{1.619}{V^{3/2}} + \frac{2.879}{V^6} \right), \qquad (3-6)$$

where: r_c is the fiber core radius and

V is the normalized frequency in the fiber, $V \equiv k r_c \sqrt{2n\Delta n}$ (where n is the index of the core material and Δn is the difference between the indices of the core and cladding).

Snyder [6] has shown that it is possible to approximate the coupling coefficient between a field in free-space and a fiber mode with an overlap integral. Using Snyder's general formula, the power coupled into the fiber is:

$$\widetilde{P}_f \simeq \left| \iint d\overline{r}\, \widetilde{\underline{U}}_f(\overline{r})\, \underline{U}^*_{11}(\overline{r}) \right|^2$$

$$= \left| \frac{1}{(j\lambda f)(\lambda L)^2} \int\int_{A_t} d\bar{\rho}\, \tilde{\underline{T}}(\bar{\rho})\, \underline{\mathfrak{U}}_T(\frac{\bar{\rho}}{\lambda L}) \int\int d\bar{r}\, \underline{U}^*_{11}(\bar{r})\, \mathcal{W}_R\left(-\frac{\bar{\rho} - \frac{L}{f}\bar{r}}{\lambda L} \right) \right|^2$$

$$= \left| \frac{1}{(j\lambda f)(\lambda L)^2} \int\int_{A_t} d\bar{\rho}\, \tilde{\underline{T}}(\bar{\rho})\, \underline{\mathfrak{U}}_T(\frac{\bar{\rho}}{\lambda L})\, \mathcal{F}_{\bar{\rho}_1}\left\{ W_R(\bar{\rho}_1)\, \underline{\mathfrak{U}}^*_{11}(-\frac{\bar{\rho}_1}{\lambda f}) \right\}\Big|_{-\frac{\bar{\rho}}{\lambda L}} \right|^2, \qquad (3-7)$$

where the second equation comes from writing out the equation for $\tilde{\underline{U}}_f()$ and rearranging the terms. The third equation comes from writing out the equation for $\mathcal{W}_R()$ and rearranging the terms. Defining the truncated or apertured version of the fiber field mode by the term:

$$\underline{\mathfrak{U}}^*_{11t}(\frac{\bar{\rho}}{\lambda L}) \equiv \mathcal{F}_{\bar{\rho}_1}\left\{ W_R(\bar{\rho}_1)\, \underline{\mathfrak{U}}^*_{11}(-\frac{\bar{\rho}_1}{\lambda f}) \right\}\Big|_{-\frac{\bar{\rho}}{\lambda L}}, \qquad (3-8)$$

and using it in the previous equation gives:

$$\tilde{P}_f \simeq \left| \frac{1}{(j\lambda f)(\lambda L)^2} \int\int_{A_t} d\bar{\rho}\, \tilde{\underline{T}}(\bar{\rho})\, \underline{\mathfrak{U}}_T(\frac{\bar{\rho}}{\lambda L})\, \underline{\mathfrak{U}}^*_{11t}(\frac{\bar{\rho}}{\lambda L}) \right|^2. \qquad (3-9)$$

This equation has a very satisfying and intuitively appealing form. It is easy to see how the form of this equation is very similar to the integral portion of equation $(2-4)$ for $\tilde{\underline{y}}(t)$ for the free space mixing system. The first term here is the target, exactly the same as the free space integral. The second term is the transmitter illumination propagated to the target plane, exactly the same as the free space integral. In the free space integral, the remaining term is the local oscillator, referenced to the receiver aperture, back propagated to the target plane. Here the last term is the fiber mode field back propagated to the receiver aperture, truncated by the receiver aperture, and then the result back propagated to the target. (The Fourier transforms come from far field propagation integrals.)

Using the target statistics of equation $(2-6)$ and taking the expectation of $\tilde{P}_f$ gives:

$$E[\tilde{P}_f] \simeq \left| \frac{1}{(j\lambda f)(\lambda L)^2} \right|^2 \int\int_{A_t} d\bar{\rho}\, \lambda^2 \mathcal{T}_s \left| \underline{\mathfrak{U}}_T(\frac{\bar{\rho}}{\lambda L})\, \underline{\mathfrak{U}}^*_{11t}(\frac{\bar{\rho}}{\lambda L}) \right|^2$$

$$= \frac{\mathcal{T}_s}{f^2(\lambda L)^4} \int\int_{A_t} d\bar{\rho} \left| \underline{\mathfrak{U}}_T(\frac{\bar{\rho}}{\lambda L}) \right|^2 \left| \underline{\mathfrak{U}}^*_{11t}(\frac{\bar{\rho}}{\lambda L}) \right|^2. \qquad (3-10)$$

The last term in the integral is:

$$\left| \underline{\mathfrak{U}}^*_{11t}(\frac{\bar{\rho}}{\lambda L}) \right|^2 = \int\int d\bar{\rho}'_1\, W_R(\bar{\rho}'_1)\, \underline{\mathfrak{U}}^*_{11}(-\frac{\bar{\rho}_1}{\lambda f})\, \exp\left(-\frac{jk}{2L} |\bar{\rho} - \bar{\rho}'_1|^2 \right) \cdot$$

$$\int\int d\bar{\rho}'_2\, W_R(\bar{\rho}'_2)\, \underline{\mathfrak{U}}_{11}(-\frac{\bar{\rho}_2}{\lambda f})\, \exp\left(\frac{jk}{2L} |\bar{\rho} - \bar{\rho}'_2|^2 \right)$$

$$\simeq \int\!\!\int d\bar{\rho}_o \int\!\!\int d\overline{\Delta\rho}\; W_R(\bar{\rho}_o + \tfrac{1}{2}\overline{\Delta\rho})\; W_R(\bar{\rho}_o - \tfrac{1}{2}\overline{\Delta\rho}) \cdot$$

$$\mathfrak{U}_{11}^{*}\left(-\frac{\bar{\rho}_o + \frac{1}{2}\overline{\Delta\rho}}{\lambda f}\right) \mathfrak{U}_{11}\left(-\frac{\bar{\rho}_o - \frac{1}{2}\overline{\Delta\rho}}{\lambda f}\right) \exp\left(-j2\pi\bar{\rho}\cdot\frac{\overline{\Delta\rho}}{\lambda L}\right), \qquad (3-11)$$

where the approximation is good for the far field and the change of variables is:

$$\bar{\rho}_o \equiv \frac{\bar{\rho}_1' + \bar{\rho}_2'}{2}, \quad \overline{\Delta\rho} \equiv \bar{\rho}_1' - \bar{\rho}_2' \quad \Rightarrow \quad \int\!\!\int d\bar{\rho}_1' \int\!\!\int d\bar{\rho}_2' = \int\!\!\int d\bar{\rho}_o \int\!\!\int d\overline{\Delta\rho}.$$

Finally, plugging equation (3 – 11) into equation (3 – 10) and doing some algebra gives the expression:

$$E[\tilde{P}_f] \simeq \frac{\mathcal{T}_s}{f^2(\lambda L)^2} \int\!\!\int d\overline{\Delta\rho}\; \mathcal{T}_{\bar{\rho}}\left\{\left|\mathfrak{U}_T\left(\frac{\bar{\rho}}{\lambda L}\right)\right|^2\right\}\Bigg|_{\frac{\overline{\Delta\rho}}{\lambda L}} \int\!\!\int d\bar{\rho}_o\; W_R(\bar{\rho}_o + \tfrac{1}{2}\overline{\Delta\rho})\; \mathfrak{U}_{11}^{*}\left(-\frac{\bar{\rho}_o + \frac{1}{2}\overline{\Delta\rho}}{\lambda f}\right) \cdot$$

$$W_R(\bar{\rho}_o - \tfrac{1}{2}\overline{\Delta\rho})\; \mathfrak{U}_{11}\left(-\frac{\bar{\rho}_o - \frac{1}{2}\overline{\Delta\rho}}{\lambda f}\right). \qquad (3-12)$$

This is about as far as the expression can go without giving the field and aperture functions some specific forms.

B. General circular aperture

Now let the apertures have the usual circular shape with diameter d_R:

$$W_R(\bar{\rho}) = \mathrm{circ}\left(\frac{\bar{\rho}}{d_R}\right) \equiv \begin{cases} 1 & |\bar{\rho}| \le d_R/2 \\ 0 & |\bar{\rho}| > d_R/2 \end{cases}, \qquad (3-13)$$

let the transmitted beam be the usual Gaussian shape, as before:

$$\underline{U}_T(\bar{\rho}) = \sqrt{\frac{2P_T}{\pi w_o^2}}\, \exp\left(-\frac{|\bar{\rho}|^2}{w_o^2}\right) \quad \Rightarrow \quad \mathfrak{U}_T\left(\frac{\bar{\rho}}{\lambda L}\right) = \sqrt{2\pi w_o^2 P_T}\, \exp\left(-\left(\frac{\pi w_o}{\lambda L}\right)^2 |\bar{\rho}|^2\right), \qquad (3-14)$$

and the fiber mode field is as shown in equation (3 – 5). Plugging all this into equation (3 – 12), simplifying, and scaling the space variables by d_R gives:

$$E[\tilde{P}_f] \simeq \frac{\mathcal{T}_s P_T (2\pi w^2) d_R^4}{f^2(\lambda L)^2} \int\!\!\int d\overline{\Delta\rho}\, \exp\left(-\frac{1}{2}\left(\frac{d_R^2}{w_o^2} + \left(\frac{\pi w d_R}{\lambda f}\right)^2\right)|\overline{\Delta\rho}|^2\right) \cdot$$

$$\int\!\!\int d\bar{\rho}_o\, \exp\left(-2\left(\frac{\pi w d_R}{\lambda f}\right)^2 |\bar{\rho}_o|^2\right) \mathrm{circ}\left(\bar{\rho}_o + \tfrac{1}{2}\overline{\Delta\rho}\right) \mathrm{circ}\left(\bar{\rho}_o - \tfrac{1}{2}\overline{\Delta\rho}\right). \qquad (3-15)$$

Now define the parameter a^2 by the relation:

$$a^2 \equiv \frac{1}{2}\cdot\left(\frac{\pi w d_R}{\lambda f}\right)^2. \tag{3-16}$$

This is a useful parameter because it appears repeatedly in the expression for $E[\widetilde{P}_f]$ and it is proportional to the fiber design parameter w/λ and inversely proportional to the system optics f-number, f/d_R. Now the expression for $E[\widetilde{P}_f]$ simplifies to:

$$E[\widetilde{P}_f] \simeq \frac{4\mathfrak{T}_s P_T d_R^2}{\pi L^2}\cdot a^2 \int\int d\overline{\Delta\rho}\, \exp\left(-\left(\frac{1}{2}\cdot\left(\frac{d_R}{w_o}\right)^2 + a^2\right)|\overline{\Delta\rho}|^2\right)\cdot$$
$$\int\int d\overline{\rho}_o \exp\left(-4a^2|\overline{\rho}_o|^2\right)\mathrm{circ}\left(\overline{\rho}_o + \tfrac{1}{2}\overline{\Delta\rho}\right)\mathrm{circ}\left(\overline{\rho}_o - \tfrac{1}{2}\overline{\Delta\rho}\right). \tag{3-17}$$

The two exponentials are just standard Gaussian functions centered at the origins of the $\overline{\rho}_o$- and $\overline{\Delta\rho}$-planes. The circ() functions, however, are unit diameter circles centered at $\pm\frac{1}{2}\overline{\Delta\rho}$ in the $\overline{\rho}_o$-plane and the $\overline{\rho}_o$-integral is just over the area of overlap of the two circ() functions. If $|\overline{\Delta\rho}| > 1$, however, the two circ() functions will not overlap at all and the entire equation is identically equal to zero. Figure 1 depicts a graph of $\mathrm{circ}\left(\overline{\rho}_o + \frac{1}{2}\overline{\Delta\rho}\right)$ and $\mathrm{circ}\left(\overline{\rho}_o - \frac{1}{2}\overline{\Delta\rho}\right)$ in the $\overline{\rho}_o$-plane for an arbitrary $\overline{\Delta\rho}$. Notice the area of overlap will always be centered at the origin of the $\overline{\rho}_o$-plane. The area of the overlap is dependent only on $|\overline{\Delta\rho}|$ and not on the angle of $\overline{\Delta\rho}$. Furthermore, since the Gaussian function in $\overline{\rho}_o$ is circulo-symmetric, the inner integral over the $\overline{\rho}_o$-plane will also be independent of the angle of $\overline{\Delta\rho}$.

Figure 2 depicts one quadrant of the $\overline{\rho}_o$-plane for the specific case where $\overline{\Delta\rho}$ lies on the y-axis, $\overline{\Delta\rho} = \left(0, -|\overline{\Delta\rho}|\right)$, so the center of the circle whose arc is depicted in figure 2 is at coordinates $(x_c, y_c) = \left(0, -|\overline{\Delta\rho}/2|\right)$. From the equation for a circle with center at (x_c, y_c) and radius r:

$$(x - x_c)^2 + (y - y_c)^2 = r^2, \tag{3-18}$$

it is easy to find values for the coordinates on the x- and y-axes of the arbitrary point on the edge of the overlap area, (x_o, y_o), shown in figure 2:

$$x_{max} = \frac{1}{2}\sqrt{1 - |\overline{\Delta\rho}|^2} \qquad y_{max} = \frac{1}{2}\left(1 - |\overline{\Delta\rho}|\right) \qquad y_o = \frac{1}{2}\left(\sqrt{1 - 4x_o^2} - |\overline{\Delta\rho}|\right). \tag{3-19}$$

Since the inner integral over the $\overline{\rho}_o$-plane does not depend on the direction of $\overline{\Delta\rho}$, it is possible to simply select a convenient direction for $\overline{\Delta\rho}$ and perform the integration for that direction. Figure 2 depicts one convenient such choice. For this choice of $\overline{\Delta\rho}$, the integral over the entire plane is just four times (since there are four quadrants) the integral over the area shown in the figure.

Now separate the integration in the $\overline{\rho}_o$-plane into x- and y-integrals, and convert the outer integral to polar coordinates to form the expression:

$$E[\widetilde{P}_f] \simeq \frac{4\mathcal{T}_s P_T d_R^2}{\pi L^2}\cdot a^2 \int\int d\overline{\Delta\rho}\, \exp\left[-\left(\frac{1}{2}\cdot\frac{d_R^2}{w_o^2}+a^2\right)|\overline{\Delta\rho}|^2\right]\cdot$$

$$4\int_0^{\frac{1}{2}\sqrt{1-|\overline{\Delta\rho}|^2}} dx\, e^{-4a^2x^2} \int_0^{\sqrt{\frac{1}{4}-x^2}-\frac{|\overline{\Delta\rho}|}{2}} dy\, e^{-4a^2y^2}$$

$$= P_T\mathcal{T}_s\left(\frac{d_R}{L}\right)^2\sqrt{\pi}\int_0^1 du\, \exp\left[-\left(\frac{1}{2}\left(\frac{d_R}{w_o}\right)^2+a^2\right)u\right]\cdot\int_0^{a\sqrt{1-u}} dx\, e^{-x^2}\mathrm{erf}\left(\sqrt{a^2-x^2}-a\sqrt{u}\right),$$

$$(3-20)$$

where erf() is the standard error function from statistics, $\mathrm{erf}(x) \equiv \frac{2}{\sqrt{\pi}}\int_0^x dy\, e^{-y^2}$. The CNR ratio is then:

$$\frac{\mathrm{CNR}_{\text{fiber}}}{\mathrm{CNR}_{\text{free-space}}} =$$

$$\frac{2}{\sqrt{\pi}}\left(\frac{d_R}{w_o}\right)^2\int_0^1 du\, \exp\left[-\left(\frac{1}{2}\left(\frac{d_R}{w_o}\right)^2+a^2\right)u\right]\int_0^{a\sqrt{1-u}} dx\, e^{-x^2}\mathrm{erf}\left(\sqrt{a^2-x^2}-a\sqrt{u}\right). \quad (3-21)$$

There are two parameters in this expression, d_R/w_o and a. Since the exp() function and the erf() function are both very smooth functions and they fall off rapidly as a function of increasing arguments, the integrals are not too difficult or time consuming to do numerically. Before performing this numerical integration, however, let's examine the two special cases of very large and very small a values.

C. Large a parameter case

If we let the a parameter be large enough that $a > d_R/w_o$, the first exponential in equation (3 – 17) will be approximately independent of d_R/w_o. Furthermore, if a is this large, the second exponential will be sufficiently concentrated near the origin of the $\overline{\rho}_o$-plane that the $\overline{\rho}_o$-integral will approximately cover the entire area of the exponential for any $|\overline{\Delta\rho}| < 1$. Then we can approximate the inner integral by a constant (the area of the exponential) when $|\overline{\Delta\rho}| < 1$ and zero elsewhere. Therefore,

$$E[\widetilde{P}_f]\Big|_{\text{large } a} \simeq \frac{4\mathcal{T}_s P_T d_R^2}{\pi L^2}\cdot a^2\int\int d\overline{\Delta\rho}\, \exp\left(-a^2|\overline{\Delta\rho}|^2\right)\cdot$$

$$\int\int d\overline{\rho}_o\, \exp\left(-4a^2|\overline{\rho}_o|^2\right)\mathrm{circ}\left(\overline{\rho}_o+\tfrac{1}{2}\overline{\Delta\rho}\right)\mathrm{circ}\left(\overline{\rho}_o-\tfrac{1}{2}\overline{\Delta\rho}\right)$$

$$\simeq \frac{4\mathcal{T}_s P_T d_R^2}{\pi L^2} \cdot a^2 \int\int d\overline{\Delta\rho}\, \exp\left(- a^2 \left| \overline{\Delta\rho} \right|^2 \right) \cdot \int\int d\overline{\rho}_o\, \exp\left(- 4a^2 \left| \overline{\rho}_o \right|^2 \right)$$

$$= \frac{\mathcal{T}_s P_T d_R^2}{L^2} \int\int d\overline{\Delta\rho}\, \exp\left(- a^2 \left| \overline{\Delta\rho} \right|^2 \right) \qquad (3-22)$$

for $|\overline{\Delta\rho}| < 1$ and zero otherwise. Now perform the integral over the unit radius circle, $|\overline{\Delta\rho}| < 1$,

$$E[\widetilde{P}_f]\Big|_{\text{large } a} = \frac{\mathcal{T}_s P_T d_R^2}{L^2} \cdot 2\pi \cdot \frac{1 - e^{-a^2}}{2a^2} \simeq \frac{\mathcal{T}_s P_T d_R^2}{L^2} \cdot \frac{\pi}{a^2} \qquad (3-23)$$

where the approximation assumes, once again, a large a so that $e^{-a^2} \ll 1$.

Finally, the improvement of the fiber system over the free space system is:

$$\frac{\text{CNR}_{\text{fiber(large } a)}}{\text{CNR}_{\text{free-space}}} = \left(\frac{d_R}{w_o a} \right)^2. \qquad (3-24)$$

Unfortunately, this final expression is greater than one only when $a < d_R/w_o$ and we derived this result for the case of $a > d_R/w_o$! Therefore, in the large a parameter limit, the fiber mixer will always under-perform the free space mixer.

D. Small a parameter case

If $a < d_R/w_o$, the first exponential in equation $(3-17)$ will be approximately independent of a. Furthermore, if a is small enough, the second exponential will be approximately equal to one and constant over the region of the $\overline{\rho}_o$-plane enclosed by the two circ() functions. In this case, the inner integral will reduce to just the area of the overlap of the two circ() functions. For the worst case of $|\overline{\Delta\rho}| = 0$ (which gives the greatest extent of overlap of the two circ() functions) and $|\overline{\rho}_o| = \frac{1}{2}$ this will be true when:

$$e^{-a^2} > 0.9 \;\Rightarrow\; a < 0.3.$$

The overlap area is just twice the area of the segment of a circle defined by an arc and its associated chord. Figure 3 is a drawing of a circle segment with the arc, chord, and other parameters labeled. For our problem, $r = 1/2$, $d = |\overline{\Delta\rho}|/2$, and $h = r - d$. Using standard mensuration formulas for circles (see for instance [7]), $K = r^2 \cos^{-1}(1 - h/r) - (r - h)\sqrt{2rh - h^2}$ and the area of overlap in terms of our variables is:

$$\mathcal{A}\left(|\overline{\Delta\rho}| \right) \equiv 2K = \frac{1}{2}\left[\cos^{-1}\left(|\overline{\Delta\rho}| \right) - |\overline{\Delta\rho}| \sqrt{1 - |\overline{\Delta\rho}|^2} \right] \qquad (3-25)$$

Now the mean value of the power propagating in the fiber is:

$$E[\tilde{P}_f]\Big|_{\text{small } a} \simeq \frac{4\mathcal{T}_s P_T d_R^2}{\pi L^2}\cdot a^2 \int\int d\overline{\Delta\rho}\, \exp\!\left(-\frac{1}{2}\left(\frac{d_R}{w_o}\right)^2 |\overline{\Delta\rho}|^2\right) \mathcal{A}(|\overline{\Delta\rho}|). \qquad (3-26)$$

Performing a change of variable on the integral, the CNR ratio is:

$$\frac{\text{CNR}_{\text{fiber(small } a)}}{\text{CNR}_{\text{free-space}}} \simeq \frac{4}{\pi}\cdot\left(\frac{d_R}{w_o}\right)^2 a^2 \int_0^1 du\, \exp\!\left(-\frac{1}{2}\left(\frac{d_R}{w_o}\right)^2 u\right) \mathcal{A}(\sqrt{u}). \qquad (3-27)$$

Choosing the minimum d_R/w_o value, 4, and performing the integration numerically gives a value of $\simeq 0.0603$ for the integral, and

$$\frac{\text{CNR}_{\text{fiber(small } a)}}{\text{CNR}_{\text{free-space}}} = 1.2285\, a^2. \qquad (3-28)$$

Here, once again, the parameter a must be larger than about 1 in order for the fiber mixer to outperform the free space mixer. But we derived this expression under the small a assumption, so the fiber mixer will under-perform the free space mixer in this region as well. However, this equation and equation $(3-24)$ together indicate the possibility that there is an intermediate range of a values where the fiber mixer could perform as well or better than the free space mixer. The only way to determine what happens at these intermediate f-numbers, though, is to perform the integration of equation $(3-21)$ numerically.

E. Numerical analysis results

Figure 4 is a plot of the CNR ratio, in dB, as a function of the a parameter for three values of d_R/w_o, 4.0 (the minimum useful value), 7.0, and 10.0. The figure includes plots of the asymptotic expressions for large and small a parameters in the regions where they are appropriate. Figure 4 demonstrates that these asymptotic approximations are quite good.

Notice the fiber system is always worse than the free space system except for a small region around an a parameter of about two where the improvement is on the order of a few dB or less. At least part of this improvement may be illusory, however. The approximations involved in forming equations $(3-5)$ and $(3-7)$ are limited in their accuracy. In fact, Marcuse [8], in plotting his figure 6, found a similar excess performance peak problem when using exactly these same two approximations. However, as the ratio d_R/w_o gets larger, the performance improvement becomes more pronounced. This may be because as d_R/w_o gets larger, the LO-received field matching, as given by equation $(2-12)$ gets worse, whereas the fiber field-received field matching may remain at its optimum point with the appropriate choice of parameter a. In effect, the choice of the fiber design provides an additional parameter to vary in order to optimize the field coupling. Unfortunately, this is more a testimony to the poor coupling for large values of d_R/w_o than a praise for the fiber mixer. In practice

there may be some applications where a large d_R/w_o ratio would be useful. An example might be a high resolution radar (hence the large d_R) which used flood light illumination of the target (hence the small w_o). However, for cases where the design keeps d_R/w_o as small as possible, it seems the free space mixer makes the best possible use of the collected backscattered power and the fiber mixer cannot significantly improve on the free space mixer's performance.

Now we will investigate what sort of fiber and optics design the optimum a value requires. Recall the a parameter is defined by:

$$a^2 \equiv \frac{1}{2} \cdot \left(\frac{\pi w d_R}{\lambda f} \right)^2 \tag{3-29}$$

and the ratio w/λ is defined by:

$$\frac{w}{\lambda} \equiv \frac{r_c}{\lambda} g_o(V) = \frac{V g_o(V)}{2\pi\sqrt{2n\Delta n}} = \frac{V g_o(V)}{2\pi(NA)} \tag{3-30}$$

where NA is the usual numerical aperture for a fiber. However, for a single mode fiber, the range of V values of interest for is just $1.5 < V < 2.4$. Within this range, the factor $V g_o(V)$ is numerically almost constant. It varies only from 2.676 (for $V = 1.5$) to 2.641 (for $V = 2.4$). If we approximate it by $V g_o(V) \simeq 2.66$, we find $w/\lambda \simeq 0.42/NA$ and we can write:

$$a \simeq \frac{0.94}{(NA)(f/\#)} \tag{3-31}$$

where $f/\# \equiv f/d_R$ is the standard physical optics f-number. The optical f-number, however, is just the inverse of twice the numerical aperture [11]. Therefore we can write:

$$a \simeq 1.9 \, \frac{(NA)_{\text{optics}}}{(NA)_{\text{fiber}}}. \tag{3-32}$$

From figure 4 we notice the optimum chose of a is about 2 for a wide range of d_R/w_o values. Our last equation, however, tells us an a value of about 2 corresponds to the point where the numerical apertures of the optics and the fiber are approximately matched. This is exactly what we would expect, however, from simple geometrical optics reasoning! Furthermore, since reasonable single mode fibers have small numerical aperture, we will require receiver optics with a reasonably large f-number in order to optimize the coupling. This is an fortuitous result since the larget the optics f-number, the easier it is to fabricate the optics with low aberrations.

As an example calculation, return to equation $(3-31)$ and use $a = 2$ as the optimal value. Then we get the best performance when

$$f/\# \simeq \frac{1}{2(NA)_{\text{fiber}}}. \tag{3-33}$$

It is common to write the numerical aperture of the fiber in the follwing form:

$$(NA)_{\text{fiber}} = \sqrt{n_1^2 - n_2^2} \simeq n_1\sqrt{2\Delta} \tag{3-34}$$

where $\Delta \equiv \frac{n_1 - n_2}{n_1}$. Reasonable values for Δ and n_1 (the fiber core index of refraction) for a single mode fiber are $\Delta = 0.003$ and $n_1 = 1.5$ which gives $NA \simeq 0.12$ and $f/\# \simeq 4.3$. Although this is not a large f-number, particularly if the focal length must be long, it is not an unreasonably small f-number either.

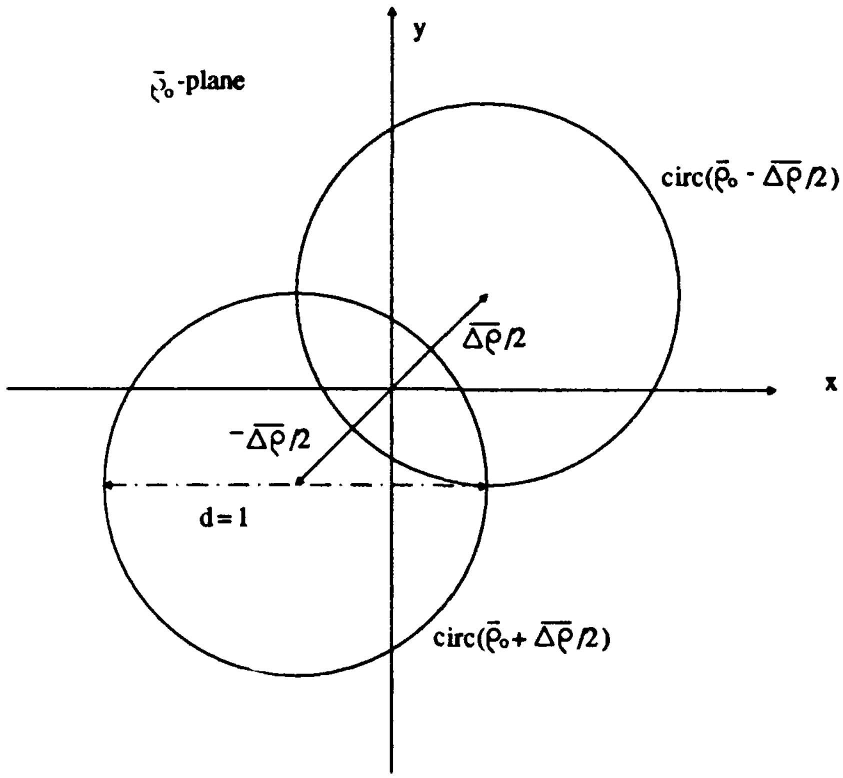


Figure 1
Circ() Function Overlap

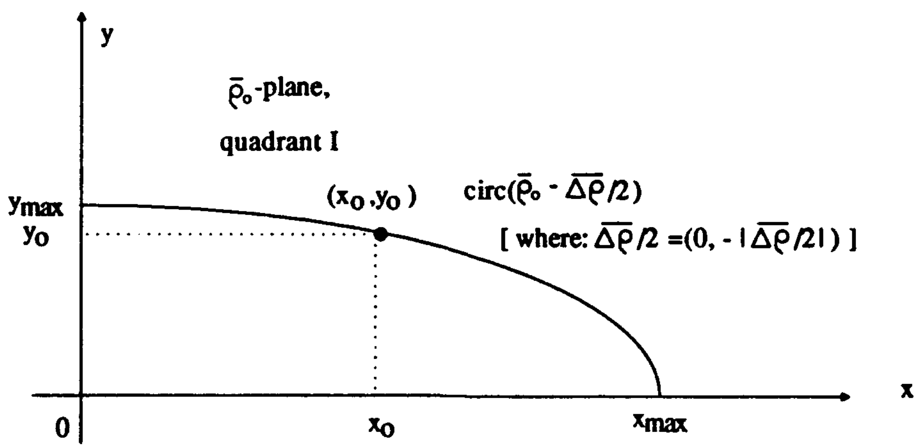


Figure 2
Circ() Function Coordinates

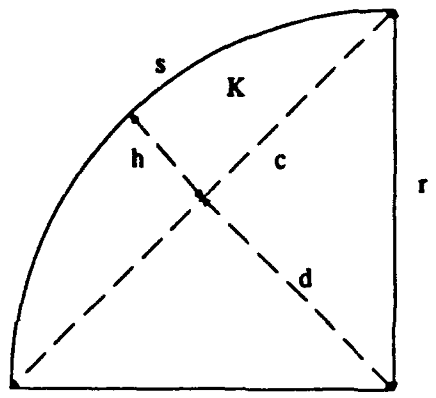
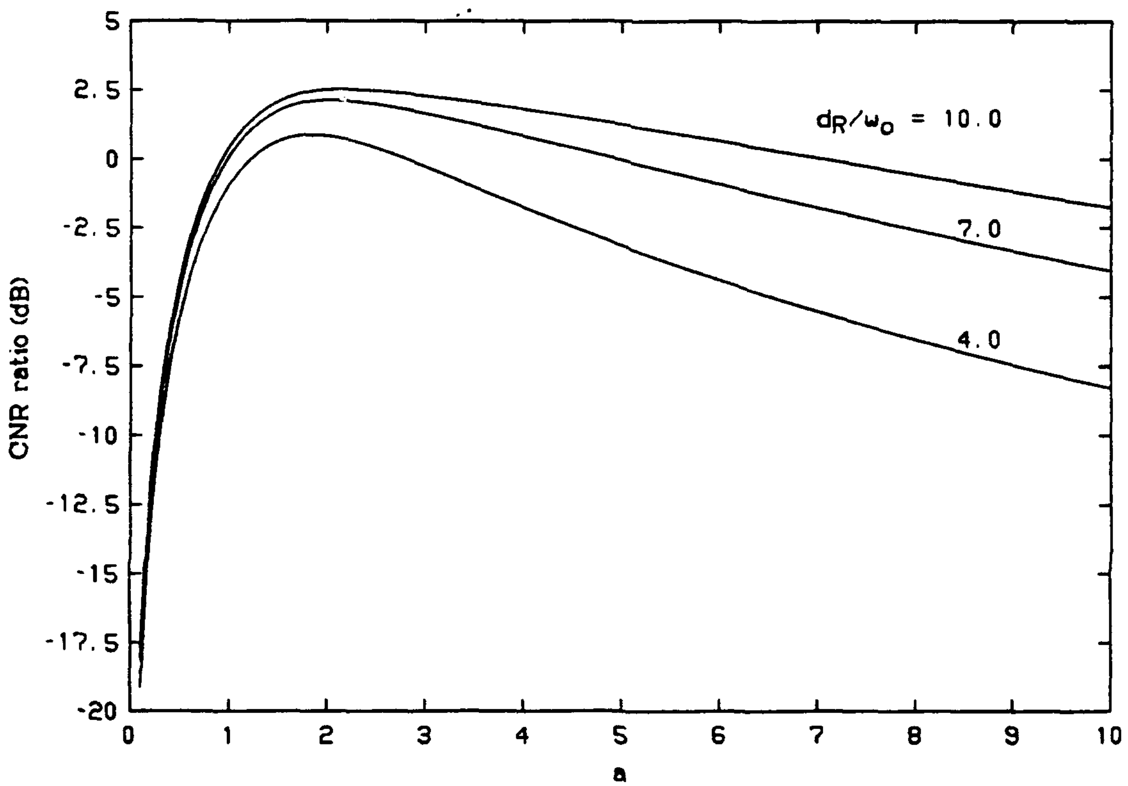


Figure 3

Circle Mensuration [7]

Figure 4
CNR ratio -vs- a parameter



IV. Summary and Conclusions

This report compared the CNR at the IF stage of a heterodyne laser radar receiver employing the traditional free space mixer with the IF CNR from a heterodyne laser radar receiver employing a fiber mixer stage. The analysis shows that the fiber mixer will achieve optimum performance when the fiber's numerical aperture is approximately matched to the receiving optics numerical aperture. This is exactly the result one would expect from simple geometric optics reasoning. When the fiber and receiving optics are optimized, the fiber mixer will still perform, at best, only marginally better than the free space mixer, however. The greatest improvement will occur in cases where the system requirements force an inefficient use of the aperture by the transmitting beam. In this case, the fiber mixer can compensate somewhat for the loss of mixing efficiency encountered with an over size aperture. Apparently the free space mixer makes nearly optimal use of the collected optical radiation even though the mixing efficiency is 25% or less.

Since single mode fibers always have quite small numerical apertures, the optimal receiving optics in a fiber mixer may have a reasonable f-number. A typical value would be around $f/\# \sim 4$. This large of an f-number may not pose too much of a problem in fabricating low aberration optics. It does, however, pose an additional constraint on the optical system design. If it is not possible to use the optimal f-number optics, it is possible to use the relations derived in this report, figure 4 in particular, to determine how much of a reduction in mixing efficiency will result. This will give a better prediction of system performance.

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Appendix I

Near Field, Far Field, and Transform Notations

The purpose of this appendix is to clarify the field and Fourier transform notations used in this report and show how this notation relates to that sometimes used in other literature, like that in reference [3].

If we define the near field distribution at the transmitter aperture plane by:

$$\underline{U}_T(\overline{\rho}, t) = \sqrt{P_T}\ \underline{s}(t)\ \underline{\xi}(\overline{\rho}) \qquad (A1-1)$$

where: P_T is the transmitted power,

$\underline{s}(t)$ is the transmitter waveform, normalized so $< |\underline{s}(t)|^2 > \ \equiv 1$, and

$\underline{\xi}(\overline{\rho})$ is the transmitter beam shape normalized so that $\int\int_{A_T} d\overline{\rho}\ |\underline{\xi}(\overline{\rho})|^2 \equiv 1$ (where A_T is the area of the transmitting aperture),

then at the target plane the far field distribution is:

$$\underline{U}_t(\overline{\rho}, t) = \sqrt{P_T}\underline{s}(t)\underline{\zeta}(\overline{\rho}) \qquad (A1-2)$$

where $\underline{\zeta}(\overline{\rho})$ is the transmitter beam shape at the target plane normalized so that $\int\int d\overline{\rho}\ |\underline{\zeta}(\overline{\rho})|^2 \equiv 1$, and is given by:

$$\underline{\zeta}(\overline{\rho}) = \int\int_{A_T} d\overline{\rho}'\ \underline{\xi}(\overline{\rho}')\ \underline{h}_L(\overline{\rho} - \overline{\rho}') \qquad (A1-3)$$

where $\underline{h}_L(\overline{\rho})$ is the free space Green's function, $\underline{h}_L(\overline{\rho}) \equiv \frac{e^{jkL}}{j\lambda L} \exp\left(\frac{jk}{2L} |\overline{\rho}|^2\right)$.

If we define the following notation to represent a 2-D spatial Fourier transform (as in the footnote of Section II):

$$\underline{\mathcal{U}}_T(\frac{\overline{\rho}}{\lambda L}, t) \equiv \mathcal{F}_{\overline{\rho}'}\{\underline{U}_T(\overline{\rho}', t)\}\Big|_{\frac{\overline{\rho}}{\lambda L}} \qquad (A1-4)$$

we can represent the far field pattern $\underline{U}_t(\overline{\rho}, t)$ in terms of $\underline{\mathcal{U}}_T(\frac{\overline{\rho}}{\lambda L}, t)$ by:

$$\underline{U}_t(\overline{\rho}, t) \simeq \frac{e^{jkL}}{j\lambda L} \exp(-\frac{jk}{2L} |\overline{\rho}|^2)\ \underline{\mathcal{U}}_T(\frac{\overline{\rho}}{\lambda L}, t) \qquad (A1-5)$$

where the approximation is good in the far field defined by $A_T \ll \lambda L$.

In the specific case of a Gaussian transmitted beam and a transmitting aperture which satisfies $d_T \geq 4w_o$ so the transmitting aperture does not truncate the beam, we have:

$$\underline{\xi}(\bar{\rho}) = \sqrt{\frac{2}{\pi w_o^2}} \exp\left(-\frac{|\bar{\rho}|^2}{w_o^2} \right),$$

$$\underline{\zeta}(\bar{\rho}) \simeq \frac{e^{jkL}}{j\lambda L} \exp\left(-\frac{jk}{2L} |\bar{\rho}|^2 \right) \sqrt{2\pi w_o^2} \exp\left(-\left(\frac{\pi w_o}{\lambda L}\right)^2 |\bar{\rho}|^2 \right), \text{ and}$$

$$\underline{\mathfrak{U}}_T\left(\frac{\bar{\rho}}{\lambda L}, t\right) = \sqrt{P_T}\underline{s}(t) \cdot \sqrt{2\pi w_o^2} \exp\left(-\left(\frac{\pi w_o}{\lambda L}\right)^2 |\bar{\rho}|^2 \right), \tag{A1-6}$$

where once again the approximation is good in the far field defined by $A_T \ll \lambda L$. Here w_o is the $1/e^2$ beam power radius, d_T is the transmitting aperture diameter, and $A_T = \pi d_T^2/4$ is still the area of the transmitting aperture.

Appendix II

Derivation of Equation 3 – 4

We will define the following field functions:

$\underline{U}_T(\overline{\rho}_1, t)$ is the transmitted field at the transmitter aperture,

$\underline{U}_t(\overline{\rho}_2, t)$ is the transmitted field propagated to the target plane (the illumination function at the target),

$\underline{\widetilde{U}}_r(\overline{\rho}_2, t)$ is the field reflected from the target,

$\underline{\widetilde{U}}_R(\overline{\rho}_3, t)$ is the field at the receiver aperture ($\underline{\widetilde{U}}_r(\overline{\rho}_2, t)$ back-propagated to the receiver aperture), and

$\underline{\widetilde{U}}_f(\overline{\rho}_f, t)$ is the field at the focal plane of the lens.

Furthermore, the free space Green's function for propagation through a distance L is:

$$\underline{h}_L(\overline{\rho}) \equiv \frac{1}{j\lambda L} e^{jkL} \exp\left(\frac{jk}{2L} |\overline{\rho}|^2\right).$$

Now we can write expressions for each of the above fields:

$$\underline{U}_t(\overline{\rho}_2, t) = \int\int_{A_T} d\overline{\rho}_1 \, \underline{U}_T(\overline{\rho}_1, t - \tfrac{L}{c}) \, \underline{h}_L(\overline{\rho}_1 - \overline{\rho}_2), \tag{A2 – 1}$$

$$\underline{\widetilde{U}}_r(\overline{\rho}_2, t) = \underline{U}_t(\overline{\rho}_2, t) \, \underline{\widetilde{T}}(\overline{\rho}_2, t), \tag{A2 – 2}$$

$$\underline{\widetilde{U}}_R(\overline{\rho}_3, t) = \int\int_{A_t} d\overline{\rho}_2 \, \underline{\widetilde{U}}_r(\overline{\rho}_2, t - \tfrac{L}{c}) \, \underline{h}_{-L}(\overline{\rho}_2 - \overline{\rho}_3), \tag{A2 – 3}$$

$$\underline{\widetilde{U}}_f(\overline{\rho}_f, t) = \int\int_{A_R} d\overline{\rho}_3 \, \underline{\widetilde{U}}_R(\overline{\rho}_2, t) \left[\frac{1}{j\lambda f} \exp\left(\frac{jk}{2f} |\overline{\rho}_f|^2\right) \cdot \exp\left(-\frac{jk}{f} \overline{\rho}_f \cdot \overline{\rho}_3\right)\right] \tag{A2 – 4}$$

where A_T, A_t, and A_R are the transmitter aperture, target area, and receiver aperture, respectively, and f is the focal length of the lens. Notice the negative range value in the Green's function in the equation for $\underline{\widetilde{U}}_R(\overline{\rho}_3, t)$, equation A2 – 3. This is because this expression is for back-propagating the reflected field to the receiver. If we keep the same coordinate system throughout, the field is now propagating in the $-z$-direction, hence the negative sign. Plugging one expression into another we get:

$$\underline{\widetilde{U}}_f(\overline{\rho}_f, t) = \int\int_{A_R} d\overline{\rho}_3 \left[\frac{1}{j\lambda f} \exp\left(\frac{jk}{2f} |\overline{\rho}_f|^2\right) \cdot \exp\left(-\frac{jk}{f} \overline{\rho}_f \cdot \overline{\rho}_3\right)\right] \cdot$$

$$\int\int_{A_t} d\overline{\rho}_2 \, \underline{h}_{-L}(\overline{\rho}_2 - \overline{\rho}_3) \, \underline{\widetilde{T}}(\overline{\rho}_2, t - \tfrac{L}{c}) \int\int_{A_T} d\overline{\rho}_1 \, \underline{U}_T(\overline{\rho}_1, t - \tfrac{2L}{c}) \, \underline{h}_L(\overline{\rho}_1 - \overline{\rho}_2)$$

$$= \int\int d\overline{\rho}_3 \, W_R(\overline{\rho}_3) \left[\frac{1}{j\lambda f} \exp\left(\frac{jk}{2f} |\overline{\rho}_f|^2\right) \cdot \exp\left(-\frac{jk}{f} \overline{\rho}_f \cdot \overline{\rho}_3\right)\right] \cdot$$

$$\int\!\!\int_{A_t} d\bar{\rho}_2\, \underline{h}_{-L}(\bar{\rho}_2-\bar{\rho}_3)\, \widetilde{\underline{T}}(\bar{\rho}_2, t-\tfrac{L}{c}) \int\!\!\int_{A_T} d\bar{\rho}_1\, \underline{U}_T(\bar{\rho}_1, t-\tfrac{2L}{c})\, \underline{h}_L(\bar{\rho}_1-\bar{\rho}_2), \qquad \text{(A2-5)}$$

where $W_R(\bar{\rho}_3)$ is the receiver aperture function and we therefore extend the limits in the $\bar{\rho}_3$-integral to the entire plane. Writing out the Green's functions and squaring the vectors in their exponential terms gives:

$$\begin{aligned}
\widetilde{\underline{U}}_f(\bar{\rho}_f, t) &= \frac{1}{(j\lambda f)(\lambda L)^2}\cdot \exp\!\left(\frac{jk}{2f}|\bar{\rho}_f|^2\right)\cdot \int\!\!\int d\bar{\rho}_3\, W_R(\bar{\rho}_3) \exp\!\left(-\frac{jk}{f}\bar{\rho}_f\cdot\bar{\rho}_3\right)\cdot \\
&\int\!\!\int_{A_t} d\bar{\rho}_2\, \widetilde{\underline{T}}(\bar{\rho}_2, t-\tfrac{L}{c}) \exp\!\left(-\frac{jk}{2L}\left(|\bar{\rho}_2|^2+|\bar{\rho}_3|^2-2\,\bar{\rho}_2\cdot\bar{\rho}_3\right)\right)\cdot \\
&\int\!\!\int_{A_T} d\bar{\rho}_1\, \underline{U}_T(\bar{\rho}_1, t-\tfrac{2L}{c}) \exp\!\left(+\frac{jk}{2L}\left(|\bar{\rho}_1|^2+|\bar{\rho}_2|^2-2\,\bar{\rho}_1\cdot\bar{\rho}_2\right)\right). \qquad \text{(A2-6)}
\end{aligned}$$

The quadratic terms in $\bar{\rho}_2$ cancel while the quadratic terms in $\bar{\rho}_1$ and $\bar{\rho}_3$ are insignificant in the far field where $A_T \ll \lambda L$ and $A_R \ll \lambda L$. Furthermore, the quadratic term in $\bar{\rho}_f$ is negligible since fiber core diameters are on the order of 5λ or less for single mode fibers. This leaves nothing but a sequence of Fourier transform integrals:

$$\begin{aligned}
\widetilde{\underline{U}}_f(\bar{\rho}_f, t) &= \frac{1}{(j\lambda f)(\lambda L)^2}\int\!\!\int d\bar{\rho}_3\, W_R(\bar{\rho}_3) \exp\!\left(-\frac{jk}{f}\bar{\rho}_f\cdot\bar{\rho}_3\right) \int\!\!\int_{A_t} d\bar{\rho}_2\, \widetilde{\underline{T}}(\bar{\rho}_2, t-\tfrac{L}{c})\,\cdot \\
&\exp\!\left(\frac{jk}{L}\bar{\rho}_2\cdot\bar{\rho}_3\right) \int\!\!\int_{A_T} d\bar{\rho}_1\, \underline{U}_T(\bar{\rho}_1, t-\tfrac{2L}{c}) \exp\!\left(-\frac{jk}{L}\bar{\rho}_1\cdot\bar{\rho}_2\right). \qquad \text{(A2-7)}
\end{aligned}$$

Under the assumption that we design the system such that the diameter of the transmitter aperture is large enough that the transmitted beam is not truncated significantly, we can extend the limits of the $\bar{\rho}_1$-integral to the entire plane. Then, rearranging terms slightly, we have:

$$\begin{aligned}
&= \frac{1}{(j\lambda f)(\lambda L)^2}\int\!\!\int_{A_t} d\bar{\rho}_2\, \widetilde{\underline{T}}(\bar{\rho}_2, t-\tfrac{L}{c}) \int\!\!\int d\bar{\rho}_1\, \underline{U}_T(\bar{\rho}_1, t-\tfrac{2L}{c}) \exp\!\left(-\frac{jk}{L}\bar{\rho}_1\cdot\bar{\rho}_2\right)\cdot \\
&\int\!\!\int d\bar{\rho}_3\, W_R(\bar{\rho}_3) \exp\!\left(-\frac{jk}{f}\bar{\rho}_f\cdot\bar{\rho}_3\right)\cdot \exp\!\left(\frac{jk}{L}\bar{\rho}_2\cdot\bar{\rho}_3\right) \\
&= \frac{1}{(j\lambda f)(\lambda L)^2}\int\!\!\int_{A_t} d\bar{\rho}_2\, \widetilde{\underline{T}}(\bar{\rho}_2, t-\tfrac{L}{c}) \int\!\!\int d\bar{\rho}_1\, \underline{U}_T(\bar{\rho}_1, t-\tfrac{2L}{c}) \exp\!\left(-j2\pi\bar{\rho}_1\cdot\frac{\bar{\rho}_2}{\lambda L}\right)\cdot \\
&\int\!\!\int d\bar{\rho}_3\, W_R(\bar{\rho}_3) \exp\!\left(-j2\pi\bar{\rho}_3\cdot\frac{\frac{L}{f}\bar{\rho}_f-\bar{\rho}_2}{\lambda L}\right),
\end{aligned}$$

which we can rewrite in our Fourier transform notation as:

$$\underset{\sim}{\widetilde{U}}_f(\overline{\rho}_f, t) = \frac{1}{(j\lambda f)(\lambda L)^2}\int\int_{A_t} d\overline{\rho}_2\, \underset{\sim}{\widetilde{T}}(\overline{\rho}_2, t - \tfrac{L}{c})\mathcal{F}\left\{\underset{\sim}{U}_T(\overline{\rho}_1, t - \tfrac{2L}{c})\right\}\Big|_{\frac{\overline{\rho}_2}{\lambda L}} \cdot \mathcal{F}\left\{W_R(\overline{\rho}_3)\right\}\Big|_{\frac{\frac{L}{f}\overline{\rho}_f - \overline{\rho}_2}{\lambda L}}$$

$$= \frac{1}{(j\lambda f)(\lambda L)^2}\int\int_{A_t} d\overline{\rho}_2\, \underset{\sim}{\widetilde{T}}(\overline{\rho}_2, t - \tfrac{L}{c})\, \underset{\sim}{\mathfrak{U}}_T\left(\frac{\overline{\rho}_2}{\lambda L}, t - \frac{2L}{c}\right) \cdot \mathcal{W}_R\left(-\frac{\overline{\rho}_2 - \frac{L}{f}\overline{\rho}_f}{\lambda L}\right). \qquad \text{(A2 - 8)}$$

This is just equation 3 – 4 of the text, but with the time dependence stated explicitly.

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